Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Group Theory

Subject Code: 5SC02GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 29/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1		**Attempt the Following questions**	**(07)**
	a.	Define: Sub Group.	**1**
	b.	Define: Cyclic group.	**1**
	c.	Define: Coset.	**1**
	d.	Define: Simple group.	**1**
	e.	Define: Normal sub group.	**1**
	f.	Let $G = (Z, +)$. Give one normal sub group of G with justification.	**2**
Q-2		**Attempt all questions**	**(14)**
	a)	State and prove necessary and sufficient condition for subgroup.	**5**
	b)	Define order of an element. Let G be a group. Prove that $o(a) = o(x^{-1}ax)$for $a, x \in G$.	**5**
	c)	If H is a subgroup of a group G then prove that the set $x^{-1}Hx$ is a subgroup of G for $x \in G$.	**4**
		OR	
Q-2		**Attempt all questions**	**(14)**
	d)	Define: Quaternion group. Prove that it is group under multiplication.	**5**
	e)	Define: Congruence relation. Prove that the congruent relation is an equivalent relation.	**5**
	f)	Prove that any two right coset of a subgroup of a group are either disjoint or identical.	**4**
Q-3		**Attempt all questions**	**(14)**
	a)	State and prove Lagrange's theorem	**5**
	b)	Let K is a subgroup and H is normal subgroup of a group G. Then prove that $K \cap H$ is normal subgroup of K.	**5**
	c)	Prove that the group of prime order is cyclic.	**4**

OR

Q-3	**a)**	Define: Center of a group. Prove that center of a group is normal subgroup of a group.	**5**
	b)	Let G be a group and H, K be two subgroup of G. Then prove that HK is a subgroup of G if and only if $HK = KH$.	**5**
	c)	Prove that a homomorphism $\phi: (G,\cdot) \to (G',*)$ is one one if and only if $Ker\ \phi = \{e\}$	**4**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: Conjugate of element in group.	**1**
	b.	Define: Partition of n.	**2**
	c.	Find center of S_3.	**2**
	d.	Define: Cycle decomposition.	**2**

Q-5		**Attempt all questions**	**(14)**
	a)	State and prove fundamental theorem of homomorphism.	**7**
	b)	For a fixed element g of a given group G. If $i_g: G \to G$ by $i_g(x) = gxg^{-1}, \forall\ x \in G$ then prove that set of all inner automorphism is normal subgroup of set of all automorphism.	**5**
	c)	Find order of $(1,3)$ in $(Z_2, +_2) \times (Z_4, +_4)$.	**2**

OR

Q-5	**a)**	Suppose G is an internal direct product of normal subgroups $N_1, N_2, \dots, N_n$ then prove that (i) $N_i \cap N_j = \{e\}, i \neq j$ (ii) If $a \in N_i, b \in N_j$ then prove that $ab = ba$.	**6**
	b)	If G_1 and G_2 are groups then prove that sets $G_1 \times \{e_2\}$, $\{e_1\} \times G_2$ are normal subgroups of $G_1 \times G_2$. Also prove that $G_1 \times \{e_2\} \cong G_1$ and $\{e_1\} \times G_2 \cong G_2$.	**5**
	c)	Prove that the order of the set of all even permutation in S_n is $\frac{n!}{2}$ for $n \geq 2$.	**3**

Q-6		**Attempt all questions**	**(14)**
	a)	State and prove Cayley's theorem.	**7**
	b)	If $o(G) = p^2$ for some prime p then prove that G is abelian.	**5**
	c)	Define: Double coset.	**2**

OR

Q-6		**Attempt all Questions**	
	a)	Let G be a finite group and p be a prime number such that $p^m \mid o(G)$, $p^{m+1} \nmid o(G)$ for some $m \geq 1$ then prove that G has a subgroup of order p^m.	**7**
	b)	Let p be a prime number and G be a group of order p^n for some $n \in N$ then prove that $Z(G) \neq \{e\}$.	**5**
	c)	Let G be a group. If $a \in Z$ then prove that $C(a) = \{a\}$.	**2**